

Contents

| | |
|--|-----------|
| Preface | ix |
| 1 Sets | 1 |
| 1.1 Basic Definitions | 3 |
| 1.2 Naming and Describing Sets | 6 |
| 1.3 Comparison Relations on Sets | 10 |
| 1.4 Set Operators | 14 |
| 1.5 Principle of Inclusion/Exclusion | 28 |
| 1.6 Exercises | 34 |
| 2 Patterns: Sequences, Summations, Mathematical Induction | 43 |
| 2.1 Sequences | 44 |
| 2.2 Describing Patterns in Sequences | 46 |
| 2.3 Summations | 56 |
| 2.4 Mathematical Induction | 60 |
| 2.4.1 First Principle of Mathematical Induction | 62 |
| 2.4.2 Examples Using Mathematical Induction | 64 |
| 2.5 Exercises | 72 |
| 3 Logic | 75 |
| 3.1 Propositional Logic | 76 |
| 3.1.1 Logical Operations | 78 |
| 3.1.2 Propositional Forms | 83 |
| 3.1.3 Parse Trees and the Operator Hierarchy (*) | 86 |
| 3.1.4 Truth Tables, Tautologies, and Contradictions | 88 |
| 3.1.5 Propositional Equivalences | 92 |
| 3.1.6 Propositional Identities | 95 |
| 3.1.7 Duality (*) | 97 |
| 3.1.8 Indirect Proofs (*) | 100 |
| 3.1.9 From English to Propositions | 103 |

| | | |
|----------|---|------------|
| 3.1.10 | Logic Circuits (*) | 105 |
| 3.2 | Predicate Logic (*) | 108 |
| 3.2.1 | Quantifiers | 110 |
| 3.2.2 | Some Rules for Using Predicates | 112 |
| 3.3 | Exercises | 115 |
| 4 | Relations | 121 |
| 4.1 | Ways to Describe Relations Between Sets | 122 |
| 4.1.1 | Using English | 123 |
| 4.1.2 | Using a Picture | 126 |
| 4.1.3 | Using a Table | 128 |
| 4.1.4 | Using the Cartesian Product | 129 |
| 4.2 | Properties of Relations | 132 |
| 4.2.1 | Reflexivity | 132 |
| 4.2.2 | Symmetry | 135 |
| 4.2.3 | Transitivity | 139 |
| 4.3 | Relational Databases | 142 |
| 4.4 | Exercises | 145 |
| 5 | Functions | 149 |
| 5.1 | What is a Function? | 151 |
| 5.2 | Functions and Relations | 156 |
| 5.3 | Properties of Functions | 162 |
| 5.4 | Function Composition | 166 |
| 5.5 | Identity and Inverse Functions | 171 |
| 5.6 | An Application: Cryptography | 180 |
| 5.6.1 | Caesar Rotation | 181 |
| 5.6.2 | Cryptography in Cyber-Commerce | 182 |
| 5.7 | More About Functions | 183 |
| 5.7.1 | Standard Mathematical Functions | 183 |
| 5.7.2 | Growth Functions | 184 |
| 5.7.3 | Functions in Program Construction | 186 |
| 5.8 | An Application: Secure Storage of Passwords | 189 |
| 5.9 | Exercises | 191 |
| 6 | Counting | 197 |
| 6.1 | Counting and How to Count | 198 |
| 6.2 | Elementary Rules for Counting | 200 |
| 6.2.1 | The Addition Rule | 200 |
| 6.2.2 | The Multiplication Rule | 202 |

| | | |
|----------|--|------------|
| 6.2.3 | Using the Elementary Rules Together | 207 |
| 6.3 | Permutations and Combinations | 209 |
| 6.3.1 | Permutations | 210 |
| 6.3.2 | Combinations | 212 |
| 6.4 | Exercises | 218 |
| 7 | Probability | 223 |
| 7.1 | Terminology and Background | 224 |
| 7.2 | Complement | 228 |
| 7.3 | Elementary Rules for Probability | 230 |
| 7.3.1 | The Elementary Addition Rule | 232 |
| 7.3.2 | The Elementary Multiplication Rule | 234 |
| 7.4 | General Rules for Probability | 237 |
| 7.4.1 | The General Addition Rule | 238 |
| 7.4.2 | The General Multiplication | 240 |
| 7.5 | Bernoulli Trials and Probability Distributions | 241 |
| 7.6 | Expected Value | 244 |
| 7.7 | Exercises | 246 |
| 8 | Algorithms | 255 |
| 8.1 | What is an Algorithm? | 255 |
| 8.2 | Applications of Algorithms | 257 |
| 8.3 | Searching and Sorting Algorithms | 258 |
| 8.3.1 | Search Algorithms | 258 |
| 8.3.2 | Sorting Algorithms | 262 |
| 8.4 | Analysis of Algorithms | 267 |
| 8.4.1 | How Do We Measure Efficiency? | 267 |
| 8.4.2 | The Time Complexity of an Algorithm | 268 |
| 8.4.3 | Analysis of Several Algorithms | 270 |
| 8.4.4 | Big-O Notation (*) | 276 |
| 8.5 | Exercises | 278 |
| 9 | Graphs | 283 |
| 9.1 | Graph Notation | 286 |
| 9.1.1 | Vertices and Edges | 286 |
| 9.1.2 | Directed and Undirected Graphs | 288 |
| 9.1.3 | Complete Graphs | 289 |
| 9.2 | Euler Trails and Circuits | 291 |
| 9.2.1 | Walks, Trails, Circuits and Cycles | 291 |
| 9.2.2 | When Can We Find Euler Trails and Circuits? | 293 |

| | | |
|---|--|------------|
| 9.3 | Weighted Graphs | 295 |
| 9.4 | Minimum Spanning Tree | 297 |
| 9.4.1 | Spanning Trees | 298 |
| 9.4.2 | Prim's Algorithm for the Minimum Spanning Tree | 299 |
| 9.5 | Matrix Notation For Graphs | 300 |
| 9.6 | Exercises | 309 |
| Appendix A Our Social Networking Example | | 315 |
| Index | | 321 |