CISC 1100/1400
Structures of Comp. Sci./Discrete Structures
Chapter 1
Sets

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Outline

- Basic definitions
- Naming and describing sets
- Comparison relations on sets
- Set operations
- Principle of Inclusion/Exclusion

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- Can use variables (usually upper case letters) to denote sets

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- *Universal set* (generally denoted U): contains all elements we might ever consider (only consider what matters)

Enumerating the elements of a set

- Order doesn't matter

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- Repetitions don't count

$$\{a,b,b\} = \{a,b\}$$

(better yet: don't repeat items in a listing of elements)

Element notation

If A is a set, then

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So

- $e \in \{a, e, i, o, u\}$
- $f \notin \{a, e, i, o, u\}$

Some well-known sets

- Pretty much standard notations:
  - $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots\}$: the set of *natural numbers* (non-negative integers).
  - $\mathbb{Z} = \{\dots -3, -2, -1, 0, 1, 2, \dots\}$: the set of all *integers*.
  - $\mathbb{Q}$: the set of all *rational numbers* (fractions).
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  - $\mathbb{R}$: the set of all *real numbers*.
- Less standard (but useful) notations:
  - $\mathbb{Z}^+$ is the set of positive integers.
  - $\mathbb{Z}^-$ is the set of negative integers.
  - $\mathbb{Z}^{\geq 0}$ is the same as $\mathbb{N}$.
  - $\mathbb{R}^{>7}$ is the set of all real numbers greater than seven.

Set builder notation

Rather than listing all the elements

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(inconvenient or essentially impossible), describe sets via a *property*

$$A = \{x : p(x) \text{ is true}\}$$

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Examples:

$$\mathbb{N} = \{\, x \mid x \in \mathbb{Z} \text{ and } x \geq 0 \,\}$$

$$\mathbb{N} = \{\, x : x \in \mathbb{Z} \text{ and } x \geq 0 \,\}$$

$$\mathbb{N} = \{\, x \in \mathbb{Z} \mid x \geq 0 \,\}$$

$$\mathbb{N} = \{\, x \in \mathbb{Z} : x \geq 0 \,\}$$

Set builder notation (cont'd)

More examples:

$$\{x \in \mathbb{Q} : 2x = 7\} =$$

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- $\subset$ vs. $\subseteq$ is somewhat like $<$ vs. $\leq$

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- **Examples:** Let

$$A = \{\text{purple}, \text{blue}, \text{orange}, \text{red}\} \qquad \text{and} \qquad B = \{\text{blue}\}.$$

Fill in the missing symbol from the set $\{\in, \notin, \subseteq, \subset, \not\subseteq, =, \neq\}$ to correctly complete each of the following statements:

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$$\text{green} \ \_\_\_\_ \ A$$

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$$\begin{array}{rcl} B & \underline{\quad\quad} & A \qquad \notin, \subseteq, \subset, \neq \\ \text{blue} & \underline{\quad\quad} & A \qquad \in, \not\subseteq, \neq \\ \text{green} & \underline{\quad\quad} & A \qquad \notin, \not\subseteq, \neq \\ A & \underline{\quad\quad} & A \end{array}$$

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$$B \;\_\_\_\_\; A \qquad \notin, \subseteq, \subset, \neq$$

$$\text{blue} \;\_\_\_\_\; A \qquad \in, \not\subseteq, \neq$$

$$\text{green} \;\_\_\_\_\; A \qquad \notin, \not\subseteq, \neq$$

$$A \;\_\_\_\_\; A \qquad \notin, \subseteq, =$$

$$\{\text{blue}, \text{purple}\} \;\_\_\_\_\; B$$

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B	____	A	$\notin, \subseteq, \subset, \neq$
blue	____	A	$\in, \not\subseteq, \neq$
green	____	A	$\notin, \not\subseteq, \neq$
A	____	A	$\notin, \subseteq, =$
$\{\text{blue}, \text{purple}\}$	____	B	$\notin, \not\subseteq, \neq$
A	____	B	

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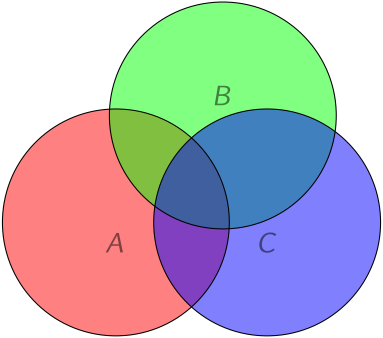
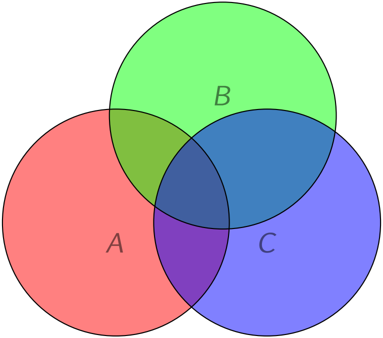
B & ___ & A & \notin, \subseteq, \subset, \neq \\
\text{blue} & ___ & A & \in, \not\subseteq, \neq \\
\text{green} & ___ & A & \notin, \not\subseteq, \neq \\
A & ___ & A & \notin, \subseteq, = \\
\{\text{blue}, \text{purple}\} & ___ & B & \notin, \not\subseteq, \neq \\
A & ___ & B & \notin, \not\subseteq, \neq
\end{array}$$

Diagram for visualizing sets and set operations



When necessary, indicate universal set via rectangle surrounding the set circles.

Venn Diagram (cont'd)



For example, might have

$$A = \{\text{Fordham students who've taken CISC 1100}\}$$
$$B = \{\text{Fordham students who've taken CISC 1600}\}$$
$$C = \{\text{Fordham students who've taken ECON 1100}\}$$

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We denote the cardinality of S by $|S|$.

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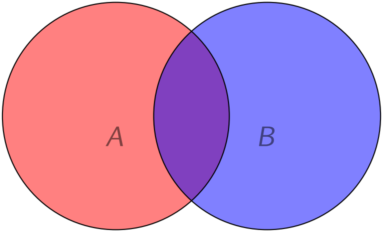
Set of all elements belonging to *either* of two given sets:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$



Set operations: Union (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 2, 4, 6, 8\}$$
$$C = \{0, 5, 10, 15\}$$

Then

$$A \cup B =$$

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Let

$$L = \{e, g, b, d, f\}$$
$$S = \{f, a, c, e\}$$

Then

$$L \cup S =$$

Set operations: Union (examples)

Let

$$\begin{aligned} A &= \{1,2,3,4,5\} \\ B &= \{0,2,4,6,8\} \\ C &= \{0,5,10,15\} \end{aligned}$$

Then

$$\begin{aligned} A \cup B &= \{0,1,2,3,4,5,6,8\} \\ B \cup A &= \{0,1,2,3,4,5,6,8\} \\ B \cup C &= \{0,2,4,5,6,8,10,15\} \\ (A \cup B) \cup C &= \{0,1,2,3,4,5,6,8,10,15\} \end{aligned}$$

Let

$$\begin{aligned} L &= \{e,g,b,d,f\} \\ S &= \{f,a,c,e\} \end{aligned}$$

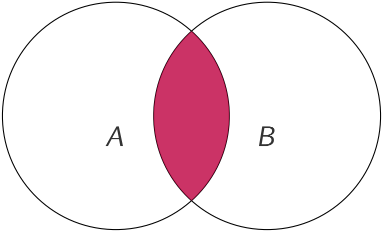
Then

$$L \cup S = \{a,b,c,d,e,f,g\}$$

Set operations: Intersection

Set of all elements belonging to *both* of two given sets:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$



Note: We say that two sets are *disjoint* if their intersection is empty.

Set operations: Intersection (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 2, 4, 6, 8\}$$
$$C = \{0, 5, 10, 15\}$$

Then

$$A \cap B =$$

Set operations: Intersection (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 2, 4, 6, 8\}$$
$$C = \{0, 5, 10, 15\}$$

Then

$$A \cap B = \{2, 4\}$$
$$B \cap A =$$

Set operations: Intersection (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
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$$A = \{1, 2, 3, 4, 5\}$$
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$$C = \{0, 5, 10, 15\}$$

Then

$$A \cap B = \{2, 4\}$$
$$B \cap A = \{2, 4\}$$
$$B \cap C = \{0\}$$
$$(A \cap B) \cap C =$$

Set operations: Intersection (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 2, 4, 6, 8\}$$
$$C = \{0, 5, 10, 15\}$$

Then

$$A \cap B = \{2, 4\}$$
$$B \cap A = \{2, 4\}$$
$$B \cap C = \{0\}$$
$$(A \cap B) \cap C = \emptyset$$

Set operations: Intersection (examples)

Let

$$\begin{aligned} A &= \{1, 2, 3, 4, 5\} \\ B &= \{0, 2, 4, 6, 8\} \\ C &= \{0, 5, 10, 15\} \end{aligned}$$

Then

$$\begin{aligned} A \cap B &= \{2, 4\} \\ B \cap A &= \{2, 4\} \\ B \cap C &= \{0\} \\ (A \cap B) \cap C &= \emptyset \end{aligned}$$

Let

$$\begin{aligned} L &= \{e, g, b, d, f\} \\ S &= \{f, a, c, e\} \end{aligned}$$

Then

$$L \cap S =$$

Set operations: Intersection (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 2, 4, 6, 8\}$$
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Then

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$$(A \cap B) \cap C = \emptyset$$

Let

$$L = \{e, g, b, d, f\}$$
$$S = \{f, a, c, e\}$$

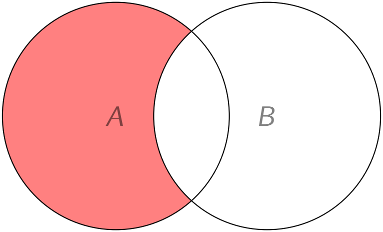
Then

$$L \cap S = \{e, f\}$$

Set operations: Difference

Set of all elements belonging to one set, but not another:

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



Set operations: Difference

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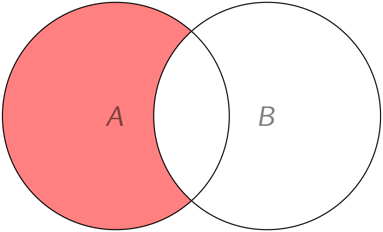

Note that

$$|A - B| = |A| - |A \cap B|$$

Set operations: Difference (examples)

Let

$$A = \{1, 2, 3, 4, 5\}$$
$$B = \{0, 2, 4, 6, 8\}$$
$$C = \{0, 5, 10, 15\}$$

Then

$$A - B =$$

Set operations: Difference (examples)

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$$B - A =$$

Set operations: Difference (examples)

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Set operations: Difference (examples)

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Then

$$A - B = \{1, 3, 5\}$$
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$$B - C = \{2, 4, 6, 8\}$$
$$C - B =$$

Set operations: Difference (examples)

Let

$$\begin{aligned} A &= \{1,2,3,4,5\} \\ B &= \{0,2,4,6,8\} \\ C &= \{0,5,10,15\} \end{aligned}$$

Then

$$\begin{aligned} A-B &= \{1,3,5\} \\ B-A &= \{0,6,8\} \\ B-C &= \{2,4,6,8\} \\ C-B &= \{5,10,15\} \\ (A-B)\cap(B-A) &= \end{aligned}$$

Set operations: Difference (examples)

Let

$$A = \{1,2,3,4,5\}$$
$$B = \{0,2,4,6,8\}$$
$$C = \{0,5,10,15\}$$

Then

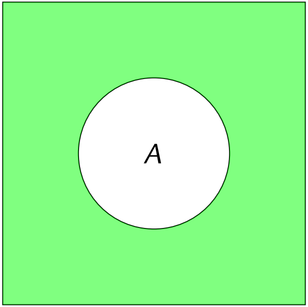
$$A - B = \{1,3,5\}$$
$$B - A = \{0,6,8\}$$
$$B - C = \{2,4,6,8\}$$
$$C - B = \{5,10,15\}$$
$$(A - B) \cap (B - A) = \emptyset \qquad \text{(Are you surprised by this?)}$$

Set operations: Complement

Set of all elements (of the universal set) that do *not* belong to a given set:

$$A' = U - A.$$

Venn diagrams dealing with complements generally use a surrounding rectangle to indicate the universal set U:



If U and A are finite sets, then

$$|A'| = |U - A| = |U| - |A|.$$

Set operations: Complement (examples)

- Let

$$U = \{\text{red}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{indigo}, \text{violet}\}$$
$$P = \{\text{red}, \text{green}, \text{blue}\}$$

Then

$$P' =$$

Set operations: Complement (examples)

- Let

$$U = \{\text{red}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{indigo}, \text{violet}\}$$
$$P = \{\text{red}, \text{green}, \text{blue}\}$$

Then

$$P' = \{\text{orange}, \text{yellow}, \text{indigo}, \text{violet}\}$$

- Let E and O respectively denote the sets of even and odd integers. Suppose that our universal set is $\mathbb{Z}$. Then

$$E' =$$

Set operations: Complement (examples)

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$$P = \{\text{red}, \text{green}, \text{blue}\}$$

  Then

$$P' = \{\text{orange}, \text{yellow}, \text{indigo}, \text{violet}\}$$

- Let E and O respectively denote the sets of even and odd integers. Suppose that our universal set is $\mathbb{Z}$. Then

$$E' = O$$
$$O' =$$

Set operations: Complement (examples)

- Let

$$U = \{\text{red}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{indigo}, \text{violet}\}$$
$$P = \{\text{red}, \text{green}, \text{blue}\}$$

  Then

$$P' = \{\text{orange}, \text{yellow}, \text{indigo}, \text{violet}\}$$

- Let E and O respectively denote the sets of even and odd integers. Suppose that our universal set is $\mathbb{Z}$. Then

$$E' = O$$
$$O' = E$$

Set operations: Power Set

Set of all subsets of a given set

$$B \in \mathscr{P}(A) \text{ if and only if } B \subseteq A$$

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Set operations: Power Set

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$$\mathscr{P}(\{a\}) =$$

Set operations: Power Set

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$$\mathscr{P}(\{a\}) = \Big\{\emptyset, \{a\}\Big\}$$

Set operations: Power Set

Set of all subsets of a given set

$$B \in \mathscr{P}(A) \text{ if and only if } B \subseteq A$$

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\mathscr{P}(\{a,b\}) &= \Big\{\emptyset, \{a\}, \{b\}, \{a,b\}\Big\} \\
\mathscr{P}(\{a,b,c\}) &=
\end{aligned}$$

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\mathscr{P}(\{a,b,c\}) &= \Big\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\Big\}
\end{aligned}$$

How many elements does $\mathscr{P}(A)$ have?

Set operations: Power Set

Set of all subsets of a given set

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\mathscr{P}(\emptyset) &= \{\emptyset\} \\
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\mathscr{P}(\{a,b,c\}) &= \Big\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\Big\}
\end{aligned}$$

How many elements does $\mathscr{P}(A)$ have?

$$|\mathscr{P}(A)| = 2^{|A|},$$

i.e.,

$$\text{if } |A| = n, \text{ then } |\mathscr{P}(A)| = 2^n.$$

Some basic laws of set theory

Here, U is a universal set, with $A, B, C, S \subseteq U$.

Name	Law
Identity	$S \cap U = S$
Identity	$S \cup \emptyset = S$
Complement	$S \cap S' = \emptyset$
Complement	$S \cup S' = U$
Double Complement	$(S')' = S$
Idempotent	$S \cap S = S$
Idempotent	$S \cup S = S$
Commutative	$A \cap B = B \cap A$
Commutative	$A \cup B = B \cup A$

Some basic laws of set theory (cont'd)

Once again, U is a universal set, with $A, B, C, S \subseteq U$.

Name	Law
Associative	$(A \cap B) \cap C = A \cap (B \cap C)$
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$
Distributive	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
DeMorgan	$(A \cap B)' = A' \cup B'$
DeMorgan	$(A \cup B)' = A' \cap B'$
Equality	$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
Transitive	if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$

- *Ordered pair*: Pair of items, in which order matters.
  - $(1, 2)$

Set operations: Cartesian Product

- *Ordered pair*: Pair of items, in which order matters.
  - $(1, 2)\ldots$ not the same thing as $(2, 1)$

Set operations: Cartesian Product

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  - $(1, 2)$... not the same thing as $(2, 1)$
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Set operations: Cartesian Product

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Set operations: Cartesian Product

- *Ordered pair*: Pair of items, in which order matters.
  - $(1, 2)$… not the same thing as $(2, 1)$
  - $(\text{red}, \text{blue})$
  - $(1, \text{green})$
- *Cartesian product* (also known as *set product*): Set of all ordered pairs from two given sets, i.e.,

$$A \times B = \{(x, y) \mid x \in A \text{ and } y \in B\}$$

Set operations: Cartesian Product (examples)

Let

$$
\begin{aligned}
A &= \{1, 2, 3\} \\
B &= \{a, b, c\} \\
C &= \{-1, 5\},
\end{aligned}
$$

Then ...

$A \times B =$

Set operations: Cartesian Product (examples)

Let

$$
\begin{aligned}
A &= \{1, 2, 3\} \\
B &= \{a, b, c\} \\
C &= \{-1, 5\},
\end{aligned}
$$

Then ...

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$$

Set operations: Cartesian Product (examples)

Let

$$A = \{1, 2, 3\}$$
$$B = \{a, b, c\}$$
$$C = \{-1, 5\},$$

Then ...

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$B \times A =$

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Then ...

$$A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c)\}$$
$$B \times A = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3)\}$$

Set operations: Cartesian Product (examples)

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Then ...

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$$C \times A = \{(-1, 1), (-1, 2), (-1, 3), (5, 1), (5, 2), (5, 3)\}$$

Set operations: Cartesian Product (examples)

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$$B \times C =$$

Set operations: Cartesian Product (examples)

Let

$$A = \{1, 2, 3\}$$
$$B = \{a, b, c\}$$
$$C = \{-1, 5\},$$

Then ...

$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$$
$$B \times A = \{(a,1),(a,2),(a,3),(b,1),(b,2),(b,3),(c,1),(c,2),(c,3)\}$$
$$C \times A = \{(-1,1),(-1,2),(-1,3),(5,1),(5,2),(5,3)\}$$
$$B \times C = \{(a,-1),(a,5),(b,-1),(b,5),(c,-1),(c,5)\}$$

Note the following:

- $A \times B \neq B \times A$ (unless $A = B$)

Set operations: Cartesian Product (examples)

Let

$$A = \{1,2,3\}$$
$$B = \{a,b,c\}$$
$$C = \{-1,5\},$$

Then ...

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$$C \times A = \{(-1,1),(-1,2),(-1,3),(5,1),(5,2),(5,3)\}$$
$$B \times C = \{(a,-1),(a,5),(b,-1),(b,5),(c,-1),(c,5)\}$$

Note the following:

- $A \times B \neq B \times A$ (unless $A = B$)
- $|A \times B| = |A| \cdot |B|$ (that's why it's called "product").

Suppose you run a fast-food restaurant. After doing a survey, you find that

- 25 people like ketchup on their hamburgers,
- 35 people like pickles on their hamburgers,
- 15 people like both ketchup *and* pickles on their hamburgers.

Principle of Inclusion/Exclusion

Suppose you run a fast-food restaurant. After doing a survey, you find that

- 25 people like ketchup on their hamburgers,
- 35 people like pickles on their hamburgers,
- 15 people like both ketchup *and* pickles on their hamburgers.

How many people like either ketchup *or* pickles (maybe both) on their hamburgers?

Principle of Inclusion/Exclusion

Suppose you run a fast-food restaurant. After doing a survey, you find that

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- 35 people like pickles on their hamburgers,
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How many people like either ketchup *or* pickles (maybe both) on their hamburgers?

Let $K = \{\text{people who like ketchup}\}$ and
$P = \{\text{people who like pickles}\}$.

Principle of Inclusion/Exclusion

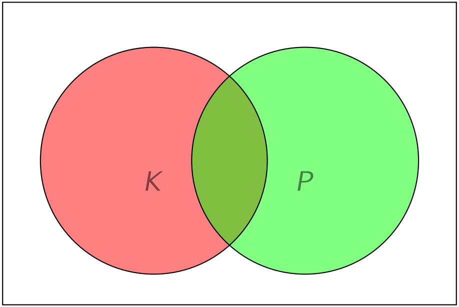
Suppose you run a fast-food restaurant. After doing a survey, you find that

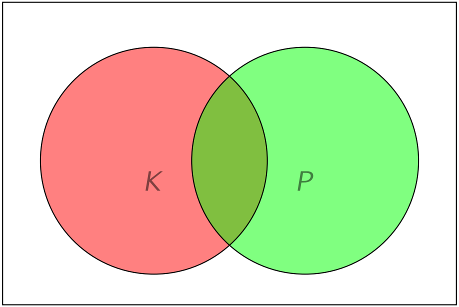
- 25 people like ketchup on their hamburgers,
- 35 people like pickles on their hamburgers,
- 15 people like both ketchup *and* pickles on their hamburgers.

How many people like either ketchup *or* pickles (maybe both) on their hamburgers?
Let $K = \{\text{people who like ketchup}\}$ and
$P = \{\text{people who like pickles}\}$. Then

$$|K| = 25 \qquad |P| = 35 \qquad |K \cap P| = 15.$$

Principle of Inclusion/Exclusion

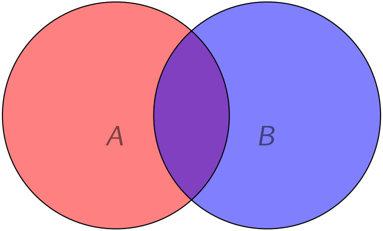




Since we don't want to count $K \cap P$ twice, we have

$$|K \cup P| = |K| + |P| - |K \cap P| = 25 + 35 - 15 = 45.$$

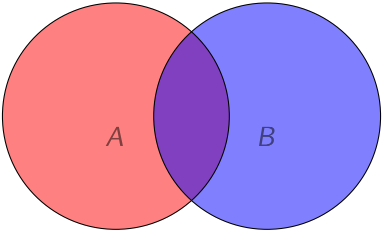
Set operations: cardinalities of union and intersection



- *Inclusion/exclusion principle:*

$$|A \cup B| =$$

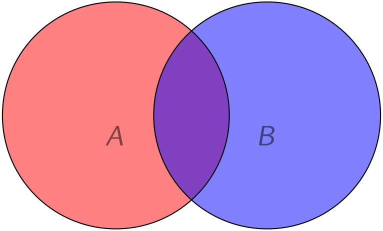
Set operations: cardinalities of union and intersection



- *Inclusion/exclusion principle:*

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Set operations: cardinalities of union and intersection



- *Inclusion/exclusion principle:*

$$|A \cup B| = |A| + |B| - |A \cap B|$$

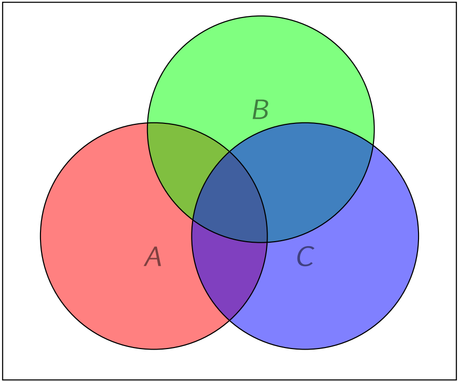
- If A and B are disjoint, then

$$|A \cup B| = |A| + |B|$$

- See the example that animates this concept.
- What about three sets (hamburger eaters who like ketchup, pickles, and tomatoes)?

Principle of Inclusion/Exclusion

For three sets:



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

Principle of Inclusion/Exclusion

Let K, P, T represent the sets of people who like ketchup, pickles, and tomatoes on their hamburgers. Suppose that

$$|K| = 20 \qquad |P| = 30 \qquad |T| = 45$$
$$|K \cap P| = 10 \qquad |K \cap T| = 12 \qquad |P \cap T| = 13$$
$$|K \cap P \cap T| = 8.$$

Let K, P, T represent the sets of people who like ketchup, pickles, and tomatoes on their hamburgers. Suppose that

$$\begin{gathered} |K| = 20 \qquad |P| = 30 \qquad |T| = 45 \\ |K \cap P| = 10 \qquad |K \cap T| = 12 \qquad |P \cap T| = 13 \\ |K \cap P \cap T| = 8. \end{gathered}$$

Then

$$\begin{aligned} |K \cup P \cup T| = |K| + |P| + |T| - \\ |K \cap P| - |K \cap T| - |P \cap T| + \\ |K \cap P \cap T| \\ = 20 + 30 + 45 - 10 - 12 - 13 + 8 \\ = 68 \end{aligned}$$