CISC 1400
Discrete Structures

Review Topics
Final Exam

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General info

- Date: Thursday 27 June, 1:00 p.m. to 3:00 p.m.
- Mainly on Chapters 5–9
- Graded on a 100-point basis, but with 110 points′ worth of questions.
- Questions based on exercises on text (either assigned or unassigned)
- One double-sided $8\frac{1}{2} \times 11$-inch sheet of notes
- Unless told otherwise, complete all arithmetic operations. You do *not* need to convert fractions (such as $\frac{1234}{5678}$) into decimals.
- You should take the practice final as part of your study.

Chapter 5: Functions

- A function $f\colon X \to Y$ is a special kind of relation on $X \times Y$.
- Terminology
  - domain
  - codomain
  - range
  - rule or description
- Composite functions
- The identity function
- Properties of a function
  - injective
  - surjective
  - bijective
- Inverse functions
  - A function is invertible iff it is bijective
  - Finding the inverse of a simple function

Chapter 6: Counting

- Using a table
- Using a tree
- Elementary rules for counting
  - addition rule
  - multiplication rule
  - using the addition and multiplication rules together
- Permutations and combinations
  - computing permutations $P(n, r)$
  - computing combinations $C(n, r)$
  - computing permutations with repetitions
- Word problems
  - Kinds of problems include
    - license plates
    - phone numbers
    - dice
    - cards
    - lotteries
    - … and so forth
  - Which rule(s) to apply?

Chapter 7: Probability

- Basic definition: $\text{Prob}(E) = |E|/|S|$ for "equally-likely" case
- Counting $|S|$, $|E|$
  - directly
  - using counting rules from Chapter 5
- Probability of complementary event

$$\text{Prob}(E') = 1 - \text{Prob}(E)$$

- Elementary rules
  - Independent and disjoint events
  - Addition rule for disjoint events

$$\text{Prob}(E_1 \cup E_2) = \text{Prob}(E_1) + \text{Prob}(E_2)$$

  - Multiplication rule for independent events:

$$\text{Prob}(E_1 \cap E_2) = \text{Prob}(E_1) \cdot \text{Prob}(E_2)$$

Chapter 7: Probability (cont'd)

- General addition rule

$$\text{Prob}(E_1 \cup E_2) = \text{Prob}(E_1) + \text{Prob}(E_2) - \text{Prob}(E_1 \cap E_2)$$

- General rules
  - General addition rule

$$\text{Prob}(E_1 \cup E_2) = \text{Prob}(E_1) + \text{Prob}(E_2) - \text{Prob}(E_1 \cap E_2)$$

  - Conditional probability

$$\text{Prob}(E_1|E_2) = \frac{\text{Prob}(E_1 \cap E_2)}{\text{Prob}(E_2)}$$

  - General multiplication rule

$$\begin{aligned}\text{Prob}(E_1 \cap E_2) &= \text{Prob}(E_1) \cdot \text{Prob}(E_2|E_1) \\ &= \text{Prob}(E_2) \cdot \text{Prob}(E_1|E_2)\end{aligned}$$

- Word problems (as before).

Chapter 7: Probability (cont'd)

- Bernoulli trials: if the probability of an event is p, then the probability of the event happening k times out of n trials is $C(n,k)p^k(1-p)^{n-k}$.
- Expected value of an event with outcomes $O_1, O_2, \ldots, O_n$ is

$$\sum_{j=1}^{n} O_j \cdot \text{Prob}(O_j) = O_1 \cdot \text{Prob}(O_1) + O_2 \cdot \text{Prob}(O_2) + \cdots + O_n \cdot \text{Prob}(O_n).$$

- Word problems
  - Kinds of problems include
    - lotteries
    - dice
    - cards
    - ... and so forth
  - Which rule to apply?

Chapter 8: Algorithms

- What is an algorithm?
- Search algorithms: know them, be able to trace on small input, know their strengths and weaknesses
  - Linear search
  - Binary search
- Sorting algorithms: know them, be able to trace on small input, know their strengths and weaknesses
  - Bubble sort
  - Merge sort
- Analysis of algorithms
  - Time complexity as a function of input size
  - Worst case, average case, best case
  - Know the complexities of search and sort algorithms we have studied, at least in terms of O-notation
- O-notation: know how to find (best) O-notation for a given function

Chapter 9: Graphs

- Three representations:
  - Graphical
  - Set-theoretic
  - Incidence matrix

  Be able to convert between them.
- Terminology
  - Vertices and edges
  - Directed vs. undirected graphs
  - Complete graphs
  - Weighted graphs
  - Walks, trails, circuits, cycles
  - Euler trails and circuits
  - Hamiltonian (Rudrata) circuits
  - Tree, spanning tree, minimal spanning tree

Chapter 9: Graphs (cont'd)

- Euler trails and circuits: existence and non-existence
- Hamiltonian (Rudrata) circuits: existence and non-existence for small cases
- Minimal spanning trees via Prim's algorithm
- Using the incidence matrix
  - Boolean matrix operations (sum, product): definition, algorithms and their cost
  - Reachability matrix: definition, algorithms and their cost